The Hyers-Ulam-Rassias Stability of Matrix Normed **Spaces**

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Abstract- In this paper, we attempt to find some stability results of duovigintic functional equation in matrix normed spaces with the help of fixed point method and we also provide an example for non-stability.

Keywords- Hyers-Ulam-Rassias stability, fixed point, duovigintic functional equation, matrix normed spaces.

1. INTRODUCTION

The stability of functional equations has emerged in relavence with a question posed by Ulam [29] in 1940. Hyers [5], brilliantly gave a partial solution for the case of the additive Cauchy functional equation for mappings between Banach spaces. This result was then improved by Aoki [1] and Rassias [18], who weakened the condition for the bound of the norm of the Cauchy difference. From 1982-1994, J. M. Rassias (see [20]- [24]) solved the Ulam problem for different mappings and for many Euler-Lagrange type quadratic mappings, by involving a product of different powers of norms. In 1994, a generalization of the Rassias theorem was obtained by Gavruta [4] by replacing the unbounded Cauchy difference by a general control function. Isac and Th. M. Rassias [6] presented some applications in non-linear analysis, especially in fixed point theory. This terminology may also be applied to the cases of other functional equations [2, 19, 28, 30]. Furthermore, the generalized Hvers-Ulam stability of functional equations and inequalities in matrix normed spaces has been studied by a number of authors, [7, 9].

Quite recently, K. Ravi and B. V. Senthil Kumar [26, 25, 27] discussed the general solution of undecic, duodecic and quattuordecic functional equations in quasi β - normed spaces. M. Nazarianpoor, J. M. Rassias and Gh. Sadeghi [17] were discovered the octadecic functional equation in multi-normed spaces. R. Murali et.al., [12]

solved the general solution and stability results in multi-Banach spaces for the following duovigintic functional equation

$$\begin{array}{r}g(a+11b)-22g(a+10b)\\+231g(a+9b)-1540g(a+8b)\\+7315g(a+7b)+74613g(a+5b)\\-170544g(a+4b)+319770g(a+3b)\\-497420g(a+2b)+646646g(a+b)\\-705432g(a)+646646g(a-b)\end{array}$$

$$-497420g(a - 2b) + 319770g(a - 3b) -170544g(a - 4b) + 74613g(a - 5b) -26334g(a - 6b) + 7315g(a - 7b) -1540g(a - 8b) + 231g(a - 9b) -22g(a - 10b) + g(a - 11b) = 1.124000728 \times 10^{21}g(b)$$
(1)

and they also discovered the other functional equations [[13]-[15]] in matrix normed spaces by using the fixed point method. In this paper, we study the generalized Hyers-Ulam-Rassias, Hyers-Ulam-Rassias, Ulam-Gavruta-Rassias and J.M. Rassias stability results for the above functional equation (1) in matrix normed spaces with the help of fixed point method. Through out this paper, let us consider $(X, \|.\|_n)$ be a matrix normed space, $(Y, \|.\|_n)$ be a matrix Banach space and let n be a fixed non-negative integer.

For a mapping $g: X \to Y$, define $\mathcal{M}g: X^2 \to Y$ by

$$\begin{split} \mathcal{M}g(a,b) &= g(a+11b) - 22g(a+10b) \\ &+ 231g(a+9b) - 1540g(a+8b) \\ &+ 7315g(a+7b) - 26334g(a+6b) \\ &+ 74613g(a+5b-170544g(a+4b)) \\ &+ 319770g(a+3b) - 497420g(a+2b) \\ &+ 646646g(a+b) - 705432g(a) \\ &+ 646646g(a-b) - 497420g(a-2b) \\ &+ 319770g(a-3b) - 170544g(a-4b) \\ &+ 74613g(a-5b) - 26334g(a-6b) \\ &+ 7315g(a-7b) - 1540g(a-8b) \\ &+ 231g(a-9b) - 22g(a-10b) \\ &+ g(a-11b) - 1.124000728 \times 10^{21}g(b) \end{split}$$

and $\mathcal{M}g_n: M_n(X^2) \to M_n(Y)$ by,

 $\mathcal{M}g([x_{rs}, y_{rs}]) = g([x_{rs} + 11y_{rs}])$ $-22g([x_{rs} + 10y_{rs}]) + 231g([x_{rs} + 9y_{rs}])$

$$+7315g([x_{rs} + 7y_{rs}]) - 26334g([x_{rs} + 6y_{rs}])$$

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 $\begin{array}{l} -1540g([x_{rs}+8y_{rs}])+74613g([x_{rs}+5y_{rs}])\\ -170544g([x_{rs}+4y_{rs}])+319770g([x_{rs}+3y_{rs}])\\ +646646g([x_{rs}-y_{rs}])-497420g([x_{rs}+2y_{rs}])\\ +646646g([x_{rs}+y_{rs}])-705432g([x_{rs}])\\ -497420g([x_{rs}-2y_{rs}])+319770g([x_{rs}-3y_{rs}])\\ -170544g([x_{rs}-4y_{rs}])+74613g([x_{rs}-5y_{rs}])\\ -26334g([x_{rs}-6([y_{rs}])+7315g([x_{rs}-7y_{rs}])\\ -1540g([x_{rs}-8y_{rs}])+231g([x_{rs}-9y_{rs}])\\ -22g([x_{rs}-10y_{rs}])+g([x_{rs}-11y_{rs}])\\ -1.124000728\times10^{21}g(y_{rs}]), \end{array}$

for all $a, b \in X$ and $x = x_{rs}$, $y = y_{rs} \in M_n(X)$.

2. STABILITY OF FUNCTIONAL EQUATION (1)

Theorem 2.1 Let $l = \pm 1$ be fixed and $\zeta: X^2 \to [0, \infty)$ be a function such that there exists a $\delta < 1$ with

$$\zeta(a,b) \le 2^{22l} \delta \zeta\left(\frac{a}{2^l}, \frac{b}{2^l}\right) \quad \forall a,b \in X.$$
(2)
and $g: X \to Y$ be a mapping satisfying

$$\|\mathcal{M}g_{n}([x_{rs}], [y_{rs}])\| \leq \sum_{r,s=1}^{n} \zeta(x_{rs}, y_{rs})$$
 (3)

 $\forall x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there exists a unique duovigintic mapping $\mathcal{D}: X \to Y$ such that

$$\begin{split} \|g_{n}([x_{rs}]) - \mathcal{D}_{n}([x_{rs}])\|_{n} &\leq \\ & \sum_{i,j=1}^{n} \frac{\delta^{\frac{1-l}{2}}}{2^{22}(1-\delta)} \zeta^{*}(x_{rs}) \qquad (4) \\ & \forall x = [x_{rs}] \in M_{n}(X), \\ \text{where } \zeta^{*}(x_{rs}) &= \frac{2}{22!} [\zeta(0,2[x_{rs}]) \\ & +\zeta(11[x_{rs}], [x_{rs}]) + 22\zeta(10[x_{rs}], [x_{rs}]) \\ +231\zeta(9[x_{rs}], [x_{rs}]) + 1540\zeta(8[x_{rs}], [x_{rs}]) + \\ 7315\zeta(7[x_{rs}], [x_{rs}]) + 26334\zeta(6[x_{rs}], [x_{rs}] + \\ 74613\zeta(5[x_{rs}], [x_{rs}]) + 170544\zeta(4[x_{rs}], [x_{rs}]) + \\ & 319770\zeta(3[x_{rs}], [x_{rs}]) + 352716(0, [x_{rs}])] \end{split}$$

Proof. For the cases l = 1 and l = -1, substituting n = 1 in (3), we obtain $||\mathcal{M}g(a,b)|| \le \zeta(a,b)$ Letting (a,b) by (11a,a) and (0,2a) in (5) respectively, and combining the two resulting inequality, we arrive at ||22g(21a) - 253g(20a) + 1540g(19a) - 7084g(18a) + 26334g(17a) - 76153g(16a) +170544g(15a) - 312455g(14a) + 497420g(13a) - 672980g(12a) + 705432g(11a) +26334g(5a) - -572033g(10a) + 497420g(9a) - 490314g(8a)+ 170544g(7a) - 245157g(6a)

$$504735g(4a) \pm \frac{22!}{2}g(2a) + 22!g(a) \Big\| \\ \leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a)$$
(6)

 $\begin{array}{l} \forall a \in X. \ \text{Considering } a = 10a \ \text{and } b = a \ \text{in (5), one} \\ \text{gets} \\ \|g(21a) - 22g(20a) + 231g(19a) - \\ 1540g(18a) + 7315g(17a) - 26334g(16a) + \\ 74613g(15a) - 170544g(14a) \\ +497420g(12a) + 26334g(4a) + 7315g(3a) \\ 646646g(11a) - 705432g(10a) + \\ 319770g(13a) - 646646g(9a) - 497420g(8a) + \\ 319770g(7a) - 170544g(6a) + \\ 74613g(5a) - 1540g(2a) - 22! g(a) \| \\ \leq \zeta(10a, a) \end{array}$

$$\begin{split} \forall a \in X. \ \text{Multiplying (7) by 22, and then combining} \\ (6) \ \text{and the resulting inequality, we arrive at} \\ \|231g(20a) - 3542g(19a) + 26796g(18a) - \\ 134596g(17a) + 503195g(16a) - \\ 1470942g(15a) + 10452926g(8a) \\ & + 3439513g(14a) - 6537520g(13a) + \\ 10270260g(12a) - 13520780g(11a) \\ & + 14947471g(10a) - 13728792g(9a) \\ & - 6864396g(7a) + 3997125g(6a) - \\ 1615152g(5a) + 74613g(4a) - 159390g(3a) - \\ \frac{22!}{2}g(2a) + 22!(23)g(a) \\ \\ & \leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a) \end{split}$$

 $\forall a \in X$. Replacing a = 9a and b = a in (5), one obtains

||g(20a) - 22g(19a) + 231g(18a) - 1540g(17a)|+7315g(16a) - 26334g(15a) + 74613g(14a)-170544g(13a) + 319770g(12a) -497420g(11a) + 646646g(10a) -705432g(9a) + 646646g(8a) - 497420g(7a) +319770g(6a) - 170544g(5a) + 74613g(4a) - $26334g(3a) + 7316g(2a) - 22!g(a) \parallel \leq \zeta(9a,a)$ (9) $\forall a \in X$. Multiplying (9) by 231, and then combining (8) and the resulting inequality, we get (5)540g(19a) - 26565g(18a) + 221144g(17a) -1186570g(16a) + 4612212g(15a)-13796090g(14a) + 32858144g(13a) -63596610g(12a) + 101383240g(11a)-134427755g(10a) + 149226000g(9a -138922300g(8a) + 108039624g(7a) -69869745g(6a) + 37780512g(5a) $-17160990g(4a) + 5923764g(3a) - \frac{22!}{2}g(2a) +$ $22! (254)g(a) \left\| \le \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a) + 231\zeta(9a,a) \right\|$

(10)

 $\forall a \in X$. Letting a = 8a and b = a in (5), one obtains ||(g(19a) - 22g(18a) + 231g(17a))||

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-1540g(16a) + 7315g(15a) - 26334g(14a) +74613g(13a) - 170544g(12a)+319770g(11a) - 497420g(10a) + 646646g(9a)-705432g(8a) + 646646g(7a) -497420g(6a) + 319770g(5a)170544g(4a) + 74614g(3a) - $26356g(2a) - 22! g(a) \| \le \zeta(8a, a)$ (11) $\forall a \in X$. Multiplying (11) by 1540, and then combining (10) and the resulting inequality, we get ||7315g(18a) - 134596g(17a) +1185030g(16a) - 6652888g(15a) +26758270g(14a) - 82045876g(13a) +199041150g(12a) - 391062560g(11a)+ 631599045g(10a)-846608840q(9a) + 947442980q(8a) -887795216g(7a) + 696157055g(6a)-454665288g(5a) +245476770g(4a) - 108981796g(3a) - $\frac{22!}{2}g(2a) + 22!(1794)g(a)$ $\leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) +$ $22\zeta(10a, a) + 231\zeta(9a, a) + 1540\zeta(8a, a)$ (12) $\forall a \in X. X a = 7a \text{ and } b = a \text{ in } (5), \text{ one obtains}$ ||g(18a) - 22g(17a) + 231g(16a) -1540g(15a) + 7315g(14a) - 26334g(13a) +74613g(12a) - 497420g(5a) -170544g(11a) +319770g(10a) - 497420g(9a) + 646646g(8a) -705432g(7a) + 646646g(6a)+319771g(4a) - 170566g(3a + 74844g(2a)) $-22! g(a) \parallel \leq \zeta(7a, a)$ (13) $\forall a \in X$. Multiplying (13) by 7315, and then combining (12) and the resulting inequality, we have $\|26334g(17a) - 504735g(16a) +$ 4612212g(15a) - 26750955g(14a) +110587334g(13a) - 346752945g(12a)+856466800g(11a) + 2792018460g(9a)-1707518505g(10a)-3782772510g(8a) + 4272439864g(7a) -4034058435g(6a) + 3183962012g(5a)-2093648095g(4a) + $1138708494g(3a) - \frac{22!}{2}g(2a) + 22!(9109)g(a)$ $\leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a) +$ $231\zeta(9a, a) + 1540\zeta(8a, a) + 7315\zeta(7a, a)$ (14) $\forall a \in X. X a = 6a \text{ and } b = a \text{ in } (5), \text{ one gets}$ ||g(17a) - 22g(16a) + 231g(15a) -1540g(14a) + 7315g(13a) - 26334g(12a) +74613g(11a) - 170544g(10a) +319770g(9a) - 497420g(8a) + 646646g(7a)-705432g(6a)

+646647g(5a) -497442g(4a) + 320001g(3a) - 172084g(2a) - $22! g(a) \| \leq \zeta(6a, a)$ (15) $\forall a \in X$. Multiplying (15) by 26334, and then combining (14) and the resulting inequality, we have ||74613g(16a) - 1470942g(15a) +13803405g(14a) - 82045876g(13a) +346726611g(12a) - 1108391942g(11a) +2783587191g(10a) - 5628804720g(9a)+9316285770g(8a) - 12756335900g(7a) +14542787850g(6a)-13844840090g(5a) + 11005989540g(4a) $-7288197840g(3a) - \frac{22!}{2}g(2a)$ + 22! (35443)g(a) $\leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a)$ $+231\zeta(9a,a) + 1540\zeta(8a,a)$ $+7315\zeta(7a,a) + 26334\zeta(6a,a)$ (16) $\forall a \in X$. Letting a = 5a and b = a in (5), one gets $\|g(16a) - 22g(15a) + 231g(14a) - 1540g(13a)$ +7315g(12a) - 26334g(11a)+74613g(10a) - 170544g(9a) +319770g(8a) - 497420g(7a) + 646647g(6a)-705454g(5a) + 646877g(4a) - 498960g(3a) $+ 327085g(2a) - 22!g(a) \parallel$ $\leq \zeta(5a,a)$ (17)

 $\forall a \in X$. Multiplying (17) by 74613 , and then combining (16) and the resulting inequality, we arrive at

 $\|170544g(15a)3432198g(14a) +$ 32858144g(13a) - 199067484g(12a)+856466800g(11a) -2783512578g(10a) + 7095994750g(9a) -14542713240*g*(8*a*) +24357662560g(7a) -33705484760g(6a) + 38791199210g(5a) -37259444060*g*(4*a*) +29940704640g(3a) - $\frac{22!}{2}g(2a) + 22!(110056)g(a)$ $\leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a)$ $+231\zeta(9a,a) + 1540\zeta(8a,a)$ $+7315\zeta(7a,a) + 26334\zeta(6a,a) + 74613\zeta(5a,a)$ (18) $\forall a \in X$. Letting a = 4a and b = a in (5), one gets ||g(15a) - 22g(14a) + 231g(13a) - 1540g(12a)|+7315g(11a) - 26334g(10a)+74613g(9a) - 170544g(8a) + 319771g(7a)-497442g(6a) + 646877g(5a)-706972g(4a) + 653961g(3a) - 523754g(2a) $-22! g(a) \parallel \leq \zeta(4a, a)$

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(19) $\forall a \in X$. Multiplying (19) by 170544, and then combining (18) and the resulting inequality, we arrive at ||319770g(14a) - 6537520g(13a) +1707593118g(10a) - 30177362860g(7a)+63570276g(12a) - 391062560g(11a)-5628804720g(9a) + 14542542700g(8a)-71529791870g(5a) + 83310388740g(4a)-81588420140g(3a)+51130263690g(6a) $+5.620003638 \times 10^{20}g(2a) - 22!(280600)g(a)$ $\leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a)$ $+ 231\zeta(9a, a) + 1540\zeta(8a, a)$ $+7315\zeta(7a, a) + 26334\zeta(6a, a) + 74613\zeta(5a, a)$ $+ 170544\zeta(4a, a)$ (20) $\forall a \in X. X a = 3a \text{ and } b = a \text{ in } (5), \text{ one gets}$ ||g(14a) - 22g(13a) + 231g(12a) -1540g(11a) + 7315g(10a) - 26334g(9a) +

$$\begin{array}{l} 74614g(8a) - 170566g(7a) + \\ 320001g(6a) - 498960g(5a) + 653961g(4a) \\ - 731766g(3a) + 721259g(2a) \\ -22! g(a) \| \leq \zeta(3a, a) \end{array}$$

(21) $\forall a \in X$. Multiplying (21) by 319770, and then combining (20) and the resulting inequality, we can get

$$\begin{split} & \|497420g(13a) - 10296594g(12a) + \\ & 101383240g(11a) - 631524432g(10a) \\ & +2792018460g(9a) - 9316776084g(8a) + \\ & 24364526960g(7a) - 51196456080g(6a) + \\ & 88022647330g(5a) - 125806720300g(4a) + \\ & 152408393700g(3a) \\ & -5.620003641 \times 10^{20}g(2a) + 22!(600370)g(a) \| \\ & \leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a) + \\ & 231\zeta(9a,a) + 1540\zeta(8a,a) + 7315\zeta(7a,a) + \\ \end{split}$$

 $26334\zeta(6a, a) + 74613\zeta(5a, a) + 170544\zeta(4a, a) + 319770\zeta(3a, a)$

(22)

$$\begin{aligned} \forall a \in X. \text{ Letting } a &= 2a \text{ and } b = a \text{ in } (5), \text{ one obtains} \\ \|g(13a) - 22g(12a) + 231g(11a) - 1540g(10a) \\ &+ 7316g(9a) - 26356g(8a) \\ &+ 74844g(7a) - 172084g(6a) + \\ 327085g(5a) - 523754g(4a) + 721259g(3a) \\ &- 875976g(2a) - 22! g(a) \| \leq \zeta(2a, a) \end{aligned} \end{aligned}$$

 $\forall a \in X$. Multiplying (23) by 497420, and then combining (22) and the resulting inequality, we arrive at

 $\|646646g(12a) - 13520780g(11a) + \\ 134502368g(10a) - 847106260g(9a) + \\ 134718994400g(4a) + 3793225436g(8a) - \\ 12864375520g(7a) + 34401567200g(6a) - \\ 74675973370g(5a) - 206360258100g(3a) - \\ 5.620003636 \times 10^{20}g(2a) + 22! (1097790)g(a) \|$

$$\leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a) + 231\zeta(9a,a) + 1540\zeta(8a,a) + 7315\zeta(7a,a)$$

 $\begin{array}{l} +26334\zeta(6a,a)+74613\zeta(5a,a)+\\ 170544\zeta(4a,a)+319770\zeta(3a,a)+\\ 497420\zeta(2a,a)\end{array}$

(24)

 $\forall a \in X. \text{ Doing } a = a \text{ and } b = a \text{ in } (5), \text{ one gets} \\ \|g(12a) - 22g(11a) + 232g(10a) - \\ 1562g(9a) + 7546g(8a) - 27874g(7a) + \\ 81928g(6a) - 196878g(5a) + \\ 394383g(4a) - 667964g(3a) + 966416g(2a) \\ - 22! g(a)\| \le \zeta(a, a)$

 $\forall a \in X$. Multiplying (25) by 646646 , and then combining (24) from the resulting inequality, we obtain

||705432g(11a) - 15519504g(10a) +162954792g(9a) - 1086365280g(8a)+5160235080g(7a) -18576846290g(6a) + 52634397820g(5a) -120307195000g(4a) + 225575990600g(3a) - $5.620003642 \times 10^{20} g(2a) + 22! (1744436) g(a) \parallel$ $\leq \frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,+231\zeta(9a,a))$ $+1540\zeta(8a, a) + 7315\zeta(7a, a) +$ $26334\zeta(6a, a) + 74613\zeta(5a, a) +$ $170544\zeta(4a, a) + 319770\zeta(3a, a)$ $+497420\zeta(2a,a) + 646646\zeta(a,a)$ (26) $\forall a \in X$. Letting a = 0 and b = a in (5), we have ||g(11a) - 22g(10a) + 231g(9a) -1540g(8+7315g(7a)-26334g(6a) +74613g(5a) - 170544g(4a) + 319770g(3a)

 $\begin{aligned} -497420g(2a) &= \frac{22!}{2}g(a) \| \leq \zeta(0,a) \\ (27) \\ \forall a \in X. \text{ Multiplying (27) by 705432 , and then} \end{aligned}$

combining (26) and the resulting inequality, one gets $\|g(2a) - 4194304g(a)\| \le \frac{2}{22!} \left[\frac{1}{2}\zeta(0,2a) + \zeta(11a,a) + 22\zeta(10a,a) + 231\zeta(9a,a) + 1540\zeta(8a,a) + 7315\zeta(7a,a) + 26334\zeta(6a,a) + 74613\zeta(5a,a) + 170544\zeta(4a,a) + 319770\zeta(3a,a) + 497420\zeta(2a,a) + 646646\zeta(a,a) + 352716(0,a)]$

Hence

$$(a) - \frac{1}{2^{22l}} g(2^{l}a) \Big\| \le \frac{\delta^{\binom{l-l}{2}}}{2^{22}} \zeta^{*}(a)$$
 (29)

 $\left\|g(a) - \frac{1}{2^{22l}}g(2^{l}a)\right\| \le \frac{\delta(2^{l})}{2^{22}}\zeta^{*}(a) \quad (29)$ for all $a \in X$. Set $\mathcal{N} = \{g: X \to Y\}$ and the generalized metric ρ on \mathcal{N} as follows:

$$\rho(g,h) = \inf\{\mu \in \mathbb{R}_+ : \|g(a) - h(a)\| \le \mu \zeta^*(c), \forall a \in X\},\$$

(28)

Claim:1 It is easy to verify that (\mathcal{E}, ρ) is a complete Generalized metric.(see [8]).

Claim:2 \mathcal{T} be a strictly contractive mapping with a lipschitz constant is less than 1

Consider the mapping
$$\mathcal{T}: \mathcal{N} \to \mathcal{N}$$
 defined by
 $\mathcal{T}g(a) = \frac{1}{2^{22l}}g(2^l b) \quad \forall \quad g \in \mathcal{N}, \quad b \in X.$ Hence
 $\|\mathcal{T}g(a) - \mathcal{T}h(a)\| = \left\|\frac{1}{2^{22l}}g(2^l a) - \frac{1}{2^{22l}}h(2^l a)\right\|$
 $\leq \delta \tau \zeta^*(a) \quad \forall g, h \in \mathcal{N}, a \in X.$

This means that \mathcal{D} is a contractive mapping with lipschitz constant $\tau < 1$. From (29), we can get

$$\rho(g, \mathcal{T}g) \leq \frac{\delta^{\left(\frac{1-\iota}{2}\right)}}{2^{22}}$$

Together Claim 1 and 2 (Theorem 2.2 in [3]), then there exists a mapping $\mathcal{D}: X \to Y$ which satisfying:

1. \mathcal{D} is a unique fixed point of \mathcal{T} , which is satisfied $\mathcal{D}(2^l a) = 2^{22l} \mathcal{D}(a) \forall a \in X$.

 $2.\rho(\mathcal{T}^k h, \mathcal{D}) \to 0$ as $k \to \infty$. This implies that $\lim_{k \to \infty} \frac{1}{2^{22kl}} h(2^{kl}b) = \mathcal{D}(a) \quad \forall \ a \in X.$

3. $\rho(g, \mathcal{D}) \leq \frac{1}{1-\delta}\rho(g, \mathcal{T}g)$, which implies the inequality

$$\|g(a) - \mathcal{D}(a)\| \le \frac{\delta^{\frac{1-l}{2}}}{2^{22}(1-\delta)}\zeta^*(c) \qquad \forall a \in X.$$
(30)

It follows from (2) and (3) that

$$\|\mathcal{MD}(b,c)\| = \lim_{k \to \infty} \frac{1}{2^{22kl}} \|\mathcal{M}f(2^{kl}b, 2^{kl}c)\| \\ \leq \lim_{k \to \infty} \frac{1}{2^{22kl}} \zeta(2^{kl}a, 2^{kl}b) = 0$$

for all $a, b \in X$. Therefore, the mapping $\mathcal{D}: X \to Y$ is duovigintic mapping. By Lemma 2.1 in [9] and (30), we can get (4) Thus $\mathcal{D}: X \to Y$ is a unique duovigintic mapping satisfying (4).

Corollary 2.2 Let $l = \pm 1$ be fixed and let t, ω be non-negative real numbers with $t \neq 22$. Let $g: X \rightarrow Y$ be a mapping satisfies

$$\|\mathcal{M}g_{n}([x_{rs}], [y_{rs}])\|_{n} \leq \sum_{r,s=1}^{n} \gamma(\|x_{rs}\|^{t} + \|y_{rs}\|^{t})$$
(31)

 $\forall x = [x_{rs}], y = [y_{rs}] \in M_n(X).$

Then there exists a unique duovigintic mapping $\mathcal{D}: X \to Y$ such that $\||a_{1}([x_{1}]) - \mathcal{D}_{1}([x_{2}])\|\|$

for all
$$x = [x_{rs}] \in M_n(X)$$
, where

$$\omega_0 = \frac{2\omega}{22!} [2743798 + (11^t) + 22(10^t) + 231(9^t) + 1540(8^t) + 7315(7^t) + 26334(6^t) + 74613(5^t) + 497420.5(2^t) + 170544(4^t) + 319770(3^t)]$$

The proof is similar to the proof of Theorem 2.1.

Corollary 2.3 Let $l = \pm 1$ be fixed and let t, ω be non-negative real numbers with $t \neq 22$. Let $g: X \rightarrow Y$ be a mapping satisfies

$$\begin{aligned} \|\mathcal{M}g_{n}([x_{rs}],[y_{rs}])\|_{n} \\ &\leq \sum_{r,s=1}^{n} \gamma(\|x_{rs}\|^{d}.\|y_{rs}\|^{e} \ (32) \end{aligned}$$

 $\forall x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there exists a unique duovigintic mapping

 $\mathcal{D}: X \to Y$ such that

$$||g_n([x_{rs}]) - \mathcal{D}_n([x_{rs}])||_n \le \sum_{r,s=1}^n \frac{\omega_0}{|2^{2^2} - 2^t|} ||x_{rs}||^t$$

for all $x = [x_{rs}] \in M_n(X)$, where

$$\omega_{0} = \frac{2\omega}{22!} [646646 + (11^{d}) + 22(10^{d}) + 231(9^{d}) + 1540(8^{d}) + 7315(7^{d}) + 26334(6^{d}) + 74613(5^{d}) + 170544(4^{d}) + 319770(3^{d}) + 497420(2^{d})]$$

The proof is similar to the proof of Theorem 2.1.

Corollary 2.4 Let $l = \pm 1$ be fixed and let t, ω be non-negative real numbers with $t \neq 22$. Let $g: X \rightarrow X$ be a mapping such that

$$\|\mathcal{M}g_{n}([x_{rs}], [y_{rs}])\|_{n} \leq \sum_{r,s=1}^{n} \gamma(\|x_{rs}\|^{d} \cdot \|y_{rs}\|^{e} + \|x_{ij}\|^{t} + \|y_{rs}\|^{t})$$
(33)

 $\forall x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there exists a unique duovigintic mapping $\mathcal{D}: X \to Y$ such that

$$||g_n([x_{rs}]) - \mathcal{D}_n([x_{rs}])||_n \le \sum_{r,s=1}^n \frac{\omega_0}{|2^{22} - 2^t|} ||x_{rs}||^t$$

for all $x = [x_{rs}] \in M_n(X)$, where

$$\omega_0 = \frac{2\omega}{22!} [3390444 + 11^t + 11^d + 22(10^t + 10^d) + 231(9^t + 9^d) + 1540(8^t + 8^d) + 7315(7^t + 7^d) + 26334(6^t + 6^d) + 6^{-1}]$$

 $\begin{array}{c} (31) \\ 74613(5^{t}+5^{d})+170544(4^{t}+4^{d})+\\ 319770(3^{t}+3^{d})+497420.5(2^{t})+497420(2^{d})] \end{array}$

The proof is similar to the proof of Theorem 2.1.

Now we will provide an example to illustrate that the functional equation (1) is not stable for t = 22 in corollary 1.

Example 2.5 *Let* ζ : $\mathbb{R} \to \mathbb{R}$ *be a function defined by*

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$$\zeta(x) = \begin{pmatrix} \omega_0 x^{22}, & if |x| < 1\\ \omega_0, & otherwise \end{pmatrix}$$

where $\omega_0 > 0$ is a constant, and define a function $g: \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \sum_{n=0}^{\infty} \frac{\zeta(2^n x)}{2^{22n}}$$

for all $x \in \mathbb{R}$. Then *h* satisfies the inequality

$$\begin{aligned} |\mathcal{M}(x,y)| &\leq \frac{(112400072800000000000)}{4194303} \\ (4194304)^2 \varepsilon(|x|^{22} + |y|^{22}) \end{aligned}$$

(34)

for all $x, y \in \mathbb{R}$. Then there does not exist a duovigintic mapping $\mathcal{D}: \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ such that

$$|g(x) - \mathcal{D}(x)| \le \lambda |x|^{22} \quad \forall x \in \mathbb{R}.$$
(35)

Proof. It is easy to see that g is bounded by $\frac{4194304\varepsilon}{4194303}$

If $|x|^{22} + |y|^{22} = 0$, then (34) is trivial. If $|x|^{22} + |y|^{22} \ge \frac{1}{2^{22}}$, then L.H.S of (34) is less than (24329020080000000)(1048576) ε .

Suppose that $0 < |x|^{22} + |c|^{22} < \frac{1}{2^{22}}$, then there exists a non-negative integer k such that

$$\frac{1}{2^{22(k+1)}} \le |b|^{22} + |c|^{22} < \frac{1}{2^{22k}},\tag{36}$$

so that

$$2^{22(k-1)}|x|^{22} < \frac{1}{2^{22}}, 2^{22(k-1)}|y|^{22} < \frac{1}{2^{22}}, \text{ and} 2^{n}(y), 2^{n}(x \pm 7y), 2^{n}(x \pm 6y), 2^{n}(x \pm 5y), 2^{n}(x \pm 4y), 2^{n}(x \pm 3y), 2^{n}(x \pm 8y), 2^{n}(x \pm 2y), 2^{n}(x \pm 9y), 2^{n}(x \pm 10y), 2^{n}(x \pm 11y), 2^{n}(x) \in (-1,1) for all $n = 0, 1, 2, ..., k - 1$. Hence$$

 $\mathcal{M}\zeta(2^n x, 2^n y) = 0$ for n = 0, 1, 2, ..., k - 1. From the definition of ζ and (36), we obtain that

$$\begin{aligned} |\mathcal{M}\zeta(x,y)| &\leq \sum_{n=0}^{\infty} \frac{1}{2^{23n}} [\mathcal{M}\zeta(2^n x, 2^n y)] \\ &\leq \frac{112400072800000000000}{4194303} \\ (4194304)^2 \varepsilon (|x|^{22} + |x|^{22}). \end{aligned}$$

Therefore, ζ satisfies (34) for all $x, y \in \mathbb{R}$. Suppose on the contrary that there exists a duovigintic mapping $\mathcal{D}: \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ satisfying (35). Then there exists a constant $c \in \mathbb{R}$ such that

 $\mathcal{D}(x) = cx^{22}$ for any $x \in \mathbb{R}$.

Thus we obtain the following inequality

$$|g(x)| \le (\lambda + |c|)|x|^{22}$$
(37)

Let $m \in \mathbb{N}$ with $m\omega_0 > \lambda + |c|$. If $b \in (0, \frac{1}{2^{m-1}})$, then $2^n x \in (0,1)$

for all n = 0, 1, 2, ..., m - 1, and for this case we get

$$\begin{aligned} \zeta(x) &= \sum_{n=0}^{\infty} \frac{\psi(2^n x)}{2^{22n}} \ge \sum_{n=0}^{m-1} \frac{\omega_0(2^n x)^{22}}{2^{22n}} \\ &= m\varepsilon x^{22} > (\lambda + |c|)|x|^{22} \end{aligned}$$

which is a contradiction to above inequality (37). Therefore the duovigintic functional equation (1) is not stable for t = 22.

Conclusion

In this investigation, we established the Hyers-Ulam-Rassias stability of the duovigintic functional equation in matrix normed spaces by using the fixed point method and also provided an example for non-(Stability.

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